

Simultaneous optical wavelength interchange with a two-dimensional second-order nonlinear photonic crystal

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We present a theoretical analysis for simultaneous optical wavelength interchange and isolation of a pair of collinear input optical signals by use of two concurrent difference-frequency-generation processes in a two-dimensional second-order nonlinear photonic crystal. We have derived a set of relations, including a general nonlinear Bragg condition, that we use to determine the parameters of the nonlinear lattice, given the input wavelengths and desired exit angles of the wavelength-interchanged outputs. © 2000 Optical Society of America

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One-dimensional quasi-phase-matching¹ schemes, such as periodic poling, provide a method for periodically reestablishing the phase relationship between collinear input and output optical waves in second-order nonlinear wavelength-conversion processes. As was shown recently by Berger,² a two-dimensional (2-D) photonic crystal of periodically reversed second-order nonlinear susceptibility [$\chi^{(2)}$] removes the collinearity requirement for the process of second-harmonic generation.

In this Letter we show that a 2-D second-order nonlinear photonic crystal provides a mechanism for performing a one-step optical carrier (i.e., wavelength) interchange between a selected pair of collinear input optical signals, such as those carried in a fiber optic wavelength-division-multiplexed communication system. The interchange can be made to occur by means of two simultaneous difference-frequency-generation (DFG) processes. The physical process takes advantage of the 2-D nature of the nonlinear photonic crystal to selectively quasi-phase match and spatially segregate the interchanged optical signal pair from the unconverted inputs; the latter procedure is essential because not all the two input signals are wavelength exchanged. It is these features that distinguish the 2-D DFG method from other wavelength translation schemes such as optical phase conjugation by four-wave mixing^{3,4} and cascaded $\chi^{(2)}$ processes.^{5,6} We also show that the choice of the pump frequency is uniquely defined by the sum of the input signal frequencies. In the absence of the pump beam, no wavelength interchange takes place. We derive the lattice parameters for the wavelength interchange as a function of input wavelengths and desired output angles and illustrate the process with an example. Applications of the wavelength-interchange process include wavelength-based cross connects, wavelength add-drop multiplexers, and spectral inverters for dispersion compensation. To the best of our knowledge, this is the first selective one-step optical carrier interchange process that simultaneously segregates the interconverted signals from the unconverted ones, thus eliminating coherent in-band cross talk.

A schematic diagram of the 2-D second-order nonlinear photonic crystal and the resultant interchange

process is shown in Fig. 1. In contrast to the conventional photonic crystal, which possesses a periodic variation in the linear electric susceptibility [$\chi^{(1)}$] for the purpose of introducing a photonic bandgap,⁷⁻⁹ the lattice shown in Fig. 1 possesses a 2-D periodic variation in $\chi^{(2)}$. The linear electric susceptibility in our proposed structure is spatially invariant. The 2-D $\chi^{(2)}$ pattern can be produced in ferroelectrics such as lithium niobate (LiNbO₃) by use of the same photolithographic and electric field poling procedures as in the one-dimensional case.¹⁰

The governing equation for a material with a dominant second-order nonlinearity is

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = \mu_0 \chi^{(2)}(\mathbf{r}) \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2}, \quad (1)$$

where $\chi^{(2)}(\mathbf{r})$ is the spatially varying instantaneous second-order nonlinearity. We assume an electric field that is composed of three time-harmonic plane waves given by

$$\mathbf{E}(\mathbf{r}, t) = 1/2 \sum_{m=1}^3 \{A_m(\mathbf{r}, t) \times \exp[i(\omega_m t - \mathbf{k}_m \cdot \mathbf{r})] + \text{c.c.}\}, \quad (2)$$

where ω_m is the angular frequency, $\mathbf{k}_m(\omega_m)$ is the wave vector of the i th optical wave, and each frequency-wave-number pair satisfies the linear dispersion relation $|\mathbf{k}_m|^2 - (n_m^2/c^2)\omega_m^2 = 0$. When Eq. (2) is substituted into Eq. (1) and a slowly varying envelope approximation is assumed, the DFG resonance condition $\omega_3 = \omega_1 - \omega_2$ yields the following set of

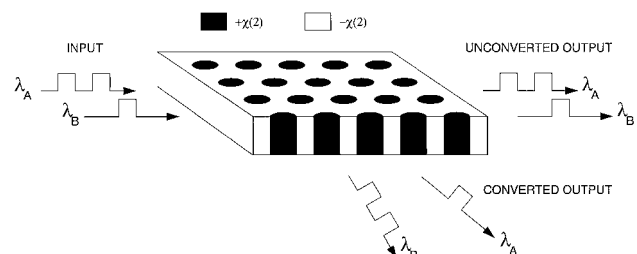


Fig. 1. Schematic diagram showing the wavelength-interchange process in a 2-D nonlinear photonic crystal.

coupled time-invariant equations (and their complex conjugates) that govern the spatial evolution of the wave amplitudes:

$$\begin{aligned}
 \mathbf{k}_1 \cdot \nabla A_1^* &= 1/2i\mu_0\chi^{(2)}(\mathbf{r})\omega_1^2 A_2^* A_3^* \\
 &\quad \times \exp[-i(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{r}], \\
 \mathbf{k}_2 \cdot \nabla A_2 &= -1/2i\mu_0\chi^{(2)}(\mathbf{r})\omega_2^2 A_1 A_3^* \\
 &\quad \times \exp[-i(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{r}], \\
 \mathbf{k}_3 \cdot \nabla A_3 &= -1/2i\mu_0\chi^{(2)}(\mathbf{r})\omega_3^2 A_1 A_2^* \\
 &\quad \times \exp[-i(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{r}]. \quad (3)
 \end{aligned}$$

The nonlinear susceptibility $\chi^{(2)}(\mathbf{r})$ can be expanded as a Fourier series: $\chi_{\text{bulk}}^{(2)} \sum_G a_G \exp(i\mathbf{G} \cdot \mathbf{r})$. The required frequency- and phase-matching conditions for normal dispersive materials can now be expressed as

$$\begin{aligned}
 \omega_3 &= \omega_1 - \omega_2, \\
 \mathbf{k}_3(\omega_3) &= \mathbf{k}_1(\omega_1) - \mathbf{k}_2(\omega_2) - \mathbf{G}, \quad (4)
 \end{aligned}$$

where \mathbf{G} is the phase-matching wave vector provided by the periodic nonlinearity $\chi^{(2)}(\mathbf{r})$ of the material. It can be shown¹¹ that, if \mathbf{a} and \mathbf{b} are the real space basis vectors in the xy plane of a 2-D periodic nonlinearity, then \mathbf{G} , a reciprocal lattice vector, is given by

$$\mathbf{G} = m\mathbf{A} + n\mathbf{B}, \quad (5)$$

where $m, n = 0, 1, 2, \dots$, $\mathbf{A} = 2\pi\mathbf{b} \times \mathbf{c}/\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$, $\mathbf{B} = 2\pi\mathbf{c} \times \mathbf{a}/\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$, and $\mathbf{c} = \mathbf{z}$.

The angle (2θ) between the input collinear waves (ω_1 and ω_2) and the output wave (ω_3) is related to the magnitudes of the wave vectors and the reciprocal lattice vector. From this relationship we have derived the following general nonlinear Bragg condition for DFG ($\omega_3 = \omega_1 - \omega_2$) as well as for sum-frequency generation (SFG; $\omega_3 = \omega_1 + \omega_2$):

$$\begin{aligned}
 \left(\frac{\lambda_3}{n_3}\right)_{\text{SFG, DFG}} &= \frac{2\pi}{|\mathbf{G}|} \left(\left\{ 1 - \left[\frac{n_1\lambda_2 \pm n_2\lambda_1}{n_3(\lambda_2 \pm \lambda_1)} \right] \right\}^2 \right. \\
 &\quad \left. + 4 \left[\frac{n_1\lambda_2 \pm n_2\lambda_1}{n_3(\lambda_2 \pm \lambda_1)} \right] \sin^2(\theta) \right)^{1/2}, \quad (6)
 \end{aligned}$$

where λ_i is the free-space wavelength and n_i is the refractive index of the material for the i th optical wave. Here \pm is replaced by $+$ for SFG and by $-$ for DFG. Equation (6) reduces to Eq. (6) of Ref. 2 for the special case of second-harmonic generation.

Consider two collinear optical data streams, $E_A(\omega_A)$ and $E_B(\omega_B)$, whose wavelengths interchange [resulting in $E_A(\omega_B)$ and $E_B(\omega_A)$]. To accomplish the interchange we use DFG with a pump field $E_p(\omega_p = \omega_A + \omega_B)$, which is also collinear with the input signals. The electric field and its subscript designate the data stream's information packet, and the angular frequency and its subscript give its carrier

identity. One can use a nonlinear version of the Ewald construction² to determine the two periodicities of the lattice that are necessary to produce the two phase-matching conditions between the pump and each of input signals A and B , given by

$$\begin{aligned}
 \omega_A &= \omega_p - \omega_B, \\
 \mathbf{k}_A'(\omega_A) &= \mathbf{k}_p(\omega_p) - \mathbf{k}_B(\omega_B) - \mathbf{G}_A, \\
 \omega_B &= \omega_p - \omega_A, \\
 \mathbf{k}_B'(\omega_B) &= \mathbf{k}_p(\omega_p) - \mathbf{k}_A(\omega_A) - \mathbf{G}_B, \quad (7)
 \end{aligned}$$

where $\mathbf{k}_A'(\omega_A)$ is the wave vector of the new outgoing optical field generated by difference-frequency mixing between the pump and the input field $E_B(\omega_B)$ and now frequency shifted to ω_A . Similarly, $\mathbf{k}_B'(\omega_B)$ is the outgoing wave vector that corresponds to a frequency shifting of data stream $E_A(\omega_A \rightarrow \omega_B)$.

To simplify the explanation, we analyze the two simultaneous processes independently. Figure 2 is a representation of the reciprocal lattice of the nonlinear 2-D direct lattice. The wave-vector difference of the two collinear optical fields $E_p(\omega_p)$ and $E_B(\omega_B)$, $\mathbf{k}_p(\omega_p) - \mathbf{k}_B(\omega_B)$, is drawn with its end point on a reciprocal lattice point. The magnitude of the wave vector for the frequency-exchanged output, $\mathbf{k}_A'(\omega_A)$, is represented as the radius of a circle (Ewald sphere in two dimensions) centered at the origin of the difference wave vector, $\mathbf{k}_p(\omega_p) - \mathbf{k}_B(\omega_B)$. The wave vectors $\mathbf{k}_A'(\omega_A)$ and $\mathbf{k}_A(\omega_A)$ are identical in magnitude and vary only in direction. If the Ewald sphere intersects a reciprocal lattice point, the reciprocal lattice vector \mathbf{G}_A completes the quasi-phase matching for the difference-frequency-generated light. The modulated light $E_B(\omega_A)$ will exit the photonic crystal along the direction defined by $\mathbf{k}_A'(\omega_A)$, different from that of the

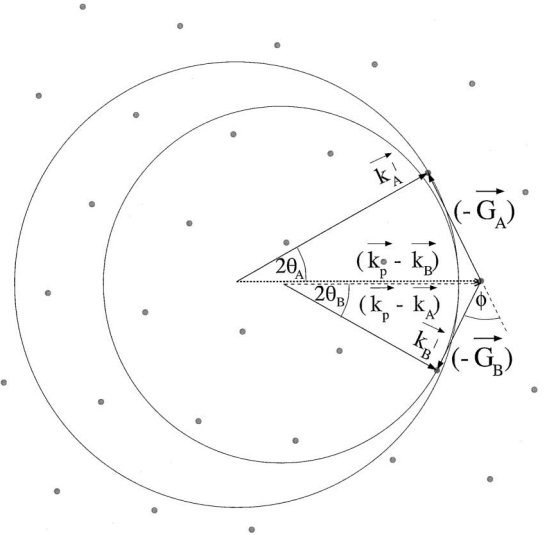


Fig. 2. Reciprocal space representation of a nonlinear lattice designed to perform wavelength interchange $\lambda_A \rightarrow \lambda_B$ and $\lambda_B \rightarrow \lambda_A$. The two Ewald spheres intersect at one common point only. The difference wave vector $\mathbf{k}_p - \mathbf{k}_A$, which overlaps $\mathbf{k}_p - \mathbf{k}_B$, is slightly displaced for purposes of visualization.

input waves and pump. An identical analysis can be done for the second input, $E_A(\omega_A) \rightarrow E_A(\omega_B)$, which is also shown in Fig. 2. It is also possible to design the lattice such that the wavelength-interchanged outputs exit the photonic crystal on the same side.

It can be shown that the real lattice spacings r_A and r_B are given by the relation

$$r_{A,B} = \frac{2\pi}{|\mathbf{G}_{A,B}|\sin(\phi)}, \quad (8)$$

where ϕ is the exterior angle subtended by \mathbf{G}_A and \mathbf{G}_B , as shown in Fig. 2. We note that in Ref. 8 (p. 117) the $\sin(\phi)$ term is omitted in the conversion from real space to reciprocal space and vice versa for nonrectangular basis vectors. We also note that in Ref. 2 the period of the reciprocal lattice is $2\pi/d$. In that case, d is the period between planes of nonlinear patterns in real space and not the real lattice period, which is equal to $d/\sin(\phi)$. Furthermore, it can be shown that the interior angle subtended by r_A and r_B is also equal to ϕ . We have derived the following expression that relates ϕ to the input wavelengths and desired exit angles:

$$\sin(\phi) = \left\{ \left[\left[\frac{|\mathbf{k}_A|}{|\mathbf{G}_A|} \sin(2\theta_A) \right]^2 - \left[\frac{|\mathbf{k}_A|}{|\mathbf{G}_A|} \sin(2\theta_A) \frac{|\mathbf{k}_B|}{|\mathbf{G}_B|} \sin(2\theta_B) \right]^2 \right]^{1/2} \pm \left[\left[\frac{|\mathbf{k}_B|}{|\mathbf{G}_B|} \sin(2\theta_B) \right]^2 - \left[\frac{|\mathbf{k}_A|}{|\mathbf{G}_A|} \sin(2\theta_A) \frac{|\mathbf{k}_B|}{|\mathbf{G}_B|} \sin(2\theta_B) \right]^2 \right]^{1/2} \right\}, \quad (9)$$

where θ_A and θ_B are the half-exit angles of the wavelength-interchanged outputs as exemplified in Fig. 2 and $\mathbf{G}_{A,B}$ are given by the general nonlinear Bragg condition [Eq. (6)]. The operator \pm is replaced by $+$ when $E_B(\omega_A)$ and $E_A(\omega_B)$ exit on opposite sides of the photonic crystal and by $-$ when they exit on the same side of the photonic crystal.

We now consider input fields at wavelengths $\lambda_A = 1550$ nm and $\lambda_B = 1550.8$ nm (100-GHz separation) whose wavelengths we wish to interchange simultaneously. An oblique nonlinear lattice in dispersive LiNbO₃ that has real lattice spacings of 5.7 and 11.8 μm , a parallelogram with an interior angle of $\phi = 43.8^\circ$, produces frequency-shifted outputs that exit the photonic crystal on opposite sides at angles of $\sim 4.5^\circ$ for \mathbf{k}_A' and $\sim 10^\circ$ for \mathbf{k}_B' . For multiple input wavelengths, exchange occurs only for those input pairs for which the lattice is designed. In the absence of the pump light, all input waves pass straight through the photonic crystal without any frequency conversion as long as the lattice is designed such that no efficient second-harmonic generation occurs for the input waves.

Because the Ewald spheres may intersect more than one reciprocal lattice point, there may be more than one

direction in which quasi-phase matching is satisfied for each of the wavelengths. If it is desired to achieve quasi-phase matching in only one direction for each of the newly mapped wavelengths, there are two ways by which this can be done. The first method is an iterative one that involves the choice of lattice periodicity. For each pair of values of \mathbf{G}_A and \mathbf{G}_B , two families of equations are generated. The intersecting points of these families of equations define all the lattice points in reciprocal space. The requirement now is to find a pair of \mathbf{G}_A and \mathbf{G}_B such that there is only one lattice point on each of the Ewald spheres. Once the pair is found, the exit angles can be determined from the general nonlinear Bragg condition given by Eq. (6) and the real space parameters from Eqs. (8) and (9). The alternative method is to manipulate the shape of the unit cell nonlinearity or equivalently the reciprocal space structure factor, $\chi_{\text{bulk}}^{(2)} a_G$, on which the efficiency of the difference-frequency-generated light depends. By manipulating the structure factor it is possible to nullify the output light in directions where light is not wanted.

In summary, we have designed a 2-D periodic $\chi^{(2)}$ photonic crystal that performs one-step, switchable data exchange between two carrier wavelengths (wavelength interchange) and have derived a set of relations, including a general nonlinear Bragg condition, that relate the input wavelengths and required exit angles to the real-space parameters of the nonlinear lattice. The device has potential applications in wavelength-sensitive routing, wavelength-layered optical cross connecting, add-drop multiplexing, and dispersion compensation. Finally, we note that a new class of nonlinear integrated optic devices may result from the incorporation of nonlinear photonic crystals into guided-wave optical circuits. This class includes any combination of simultaneous SFG and DFG processes for which Eqs. (6), (8), and (9) are the fundamental governing relations.

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